JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 30, No. 3, May–June 2007

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Near Earth Object Orbit Modification Using Gravitational Coupling

Colin R. McInnes*
University of Strathclyde,
Glasgow, G1 1XJ Scotland, United Kingdom

DOI: 10.2514/1.25864

Introduction

IT HAS been well documented that the population of near Earth objects (NEO) poses a terrestrial impact hazard [1–4]. Although current efforts are focused on detecting and cataloging such objects, various schemes for hazard mitigation have been proposed and investigated in some detail [5–15]. Nuclear devices appear attractive for deflecting potentially hazardous NEOs [5], although serious political issues arise concerning the deployment of such devices in space [11]. To overcome such difficulties, a range of nonnuclear options have been proposed. Concepts include focusing solar radiation onto the target asteroid with a large collector and smaller steerable secondary mirror to generate a hot jet of exhaust gas [6,7] or coating the asteroid to alter its albedo, and hence modify the Yarkovsky induced acceleration [8]. Somewhat more conventional approaches center on the use of kinetic energy impacts from either prograde [12,13] or retrograde orbits [9,10], or the use of continuous low thrust to increase the predicted Earth miss distance of the NEO using large solar or nuclear electric vehicles [14,15].

Recent work by Lu and Love [16] proposed the use of gravitational coupling as a novel means of modifying the orbit of an NEO using a spacecraft with continuous low thrust propulsion. In their scheme the spacecraft hovers in a static equilibrium near the NEO surface, which then induces an acceleration in the center of mass of the spacecraft-NEO system due to the stream of momentum transported by the plume of escaping exhaust gas. The control of such hovering equilibria has previously been investigated by Broschart and Scheeres [17]. This scheme has the advantage that the spacecraft is not in physical contact with the NEO and so alleviates problems with mechanical coupling of the spacecraft and NEO surface. The issue of coupling to an NEO with uncertain surface properties and internal composition is clearly problematic and has been noted by various authors [18]. The concept of gravitational coupling has been discussed previously for somewhat larger scale orbit modification scenarios which exploit the same physical principle [19,20].

This engineering Note investigates the gravitational coupling of a spacecraft to an NEO in some detail and proposes an apparently more efficient scheme to couple the spacecraft to the NEO in certain circumstances. Rather than a static equilibrium, which requires that the spacecraft exhaust plume be canted to avoid plume impingement on the NEO surface, a displaced, highly non-Keplerian orbit is used. Such an orbit will still induce an acceleration in the center of mass of the spacecraft-NEO system, but with a suitable orbit radius and displacement distance the exhaust plume need not be canted, allowing a potentially more efficient transfer of momentum to the center of mass of the spacecraft-NEO system. As with the analysis of Lu and Love [16], the higher order harmonics of the NEO potential will not be considered, nor will the effect of solar radiation pressure. Similarly, three-body dynamics will be used to investigate the acceleration of the center of mass of the spacecraft-NEO system, whereas a two-body analysis will be used to investigate coupling of the spacecraft to the NEO. Although these assumptions limit the scope of the analysis, they serve to clarify the illustration of the underlying concept.

Center-of-Mass Dynamics

To investigate the use of gravitational coupling to modify the orbit of an NEO, the dynamics of the center of mass of the spacecraft-NEO system C will be considered, as shown in Fig. 1. An inertial frame of reference $I[X, Y, Z]$ is defined, with the sun of the mass m_O at O. It is assumed that the barycenter of the sun-NEO system is located at O. The spacecraft of mass m_S and NEO of mass m_N are located at position $\mathbf{r}_S$ and $\mathbf{r}_N$ with the center of mass of the spacecraft-NEO system C located at $\mathbf{r}_C$. The spacecraft is initially assumed to be in a static equilibrium relative to the NEO such that $|\dot{\mathbf{r}}| = 0$, where $\mathbf{r}$ denotes the relative position of the NEO and spacecraft. Later, a potentially more efficient means of coupling the spacecraft and NEO will be considered using displaced non-Keplerian orbits.

The spacecraft exerts a force $\boldsymbol{f}$ due to propulsive thrust, where the force is directed along a unit vector $\mathbf{e}_N$. This vector can also compensate for the effect of solar radiation pressure acting on the spacecraft by choosing the thrust vector such that the sum of the thrust and solar radiation pressure is directed along $\boldsymbol{f}$. The equation of motion of the spacecraft may now be written as

$$m_S \ddot{\mathbf{r}}_S = -\frac{G m_O m_S}{r_S^3}\mathbf{r}_S + \frac{G m_S m_N}{r^3}\mathbf{r} + \boldsymbol{f} \tag{1}$$

where again the sun is assumed to be fixed at the origin O of the inertial frame of reference and $\mathbf{r} = \mathbf{r}_N - \mathbf{r}_S$. Similarly, the equation of motion for the NEO may be written as

$$m_N \ddot{\mathbf{r}}_N = -\frac{G m_O m_N}{r_N^3}\mathbf{r}_N - \frac{G m_S m_N}{r^3}\mathbf{r} \tag{2}$$

where the last term of Eq. (2) represents the gravitational force of the spacecraft exerted on the NEO. It is this term which is key to providing the gravitational coupling of the spacecraft and NEO. To investigate the dynamics of the center of mass C of the NEO-spacecraft system, its location is defined as

$$\mathbf{r}_C = \frac{m_S \mathbf{r}_S + m_N \mathbf{r}_N}{m_S + m_N} \tag{3}$$

Therefore, adding Eqs. (1) and (2) and using the definition of the center of mass of the spacecraft-NEO system yields

Received 14 June 2006; revision received 4 December 2006; accepted for publication 4 December 2006.

*Professor of Engineering Science, Department of Mechanical Engineering; colin.mcinnes@strath.ac.uk. Member AIAA.

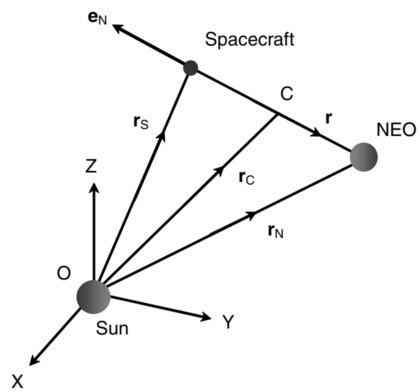

Fig. 1 Sun-NEO-spacecraft three-body system.

$$(m_S + m_N)\ddot{\mathbf{r}}_C = -\frac{Gm_O m_S}{r_S^3}\mathbf{r}_S - \frac{Gm_O m_N}{r_N^3}\mathbf{r}_N + \boldsymbol{f} \tag{4}$$

where the symmetric coupling terms have now vanished. Because the spacecraft will be located in close proximity to the NEO it is clear that $\mathbf{r}_S \approx \mathbf{r}_N \approx \mathbf{r}_C$ so that

$$(m_S + m_N)\ddot{\mathbf{r}}_C \approx -Gm_O(m_S + m_N)\frac{\mathbf{r}_C}{r_C^3} + \boldsymbol{f} \tag{5}$$

Furthermore, because $m_S \ll m_N$ an equation of motion for the center of mass of the spacecraft-NEO system C is then obtained as

$$\ddot{\mathbf{r}}_C + Gm_O\frac{\mathbf{r}_C}{r_C^3} \approx \frac{1}{m_N}\boldsymbol{f} \tag{6}$$

It can be seen that Eq. (6) has the form of a two-body equation of motion for the orbit of C about O with the addition of a forcing term from the spacecraft thrust. Because the forcing term is scaled by m_N^{-1}, the center of mass of the NEO-spacecraft system will experience an extremely small, but secular perturbation [19]. To modify the semimajor axis of the NEO the thrust $\boldsymbol{f}$ can be directed parallel to the NEO velocity vector modifying the NEO orbit period and so increasing the Earth miss distance [7,15].

Static Equilibrium

The simplest scheme to gravitationally couple the spacecraft to the NEO is through an artificial static equilibrium generated by the thrust $\boldsymbol{f}$, where again the thrust may compensate for the effect of solar radiation pressure. In this scenario the spacecraft thrust exactly balances the local gravitational force, while the momentum transported by the spacecraft exhaust plume is balanced by the momentum gained by the center of mass of the spacecraft-NEO system. For an NEO gravitational potential V the equation of motion of the spacecraft relative to the NEO is given by

$$\ddot{\mathbf{r}} = -\nabla V + \frac{1}{m_S}\boldsymbol{f} \tag{7}$$

Although active hovering of a spacecraft near an NEO with a complex potential has been considered elsewhere [17], a simpler inverse square interaction will be considered using

$$V = -\frac{Gm_N}{r} + U \tag{8}$$

where U represents the higher order harmonics of the potential. Following the analysis of Lu and Love [16], U will be neglected, although it should be noted that Broschart and Scheeres have demonstrated the possibility of stationkeeping in proximity to an irregular small body, while neglecting solar radiation pressure and the solar tide [17]. A static equilibrium is therefore obtained when the spacecraft thrust vector $\mathbf{e}_N$ is directed along the radial vector $\mathbf{e}_Z$, as shown in Fig. 2. This condition is given simply by

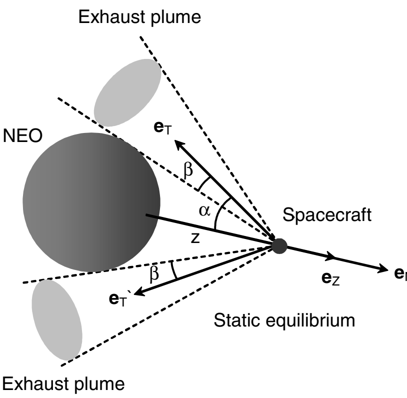

Fig. 2 Spacecraft in a static equilibrium at distance z with thrusters canted to enforce plume impingement constraints on the NEO surface.

$$\boldsymbol{f} = \frac{Gm_S m_N}{z^2}\mathbf{e}_Z \tag{9}$$

It is clear that autonomous navigation and orbit control would be required for the entire duration of the entire deflection maneuver, which would typically be of the order of 10 years [16]. It can be shown, using a two-body model, that such static equilibria are not controllable using thrust modulation alone. However, displaced non-Keplerian orbits are in principle controllable using thrust modulation only [21].

Displaced Orbits

It will now be shown that a potentially more efficient means of gravitationally coupling the spacecraft to the NEO can be found through the use of displaced highly non-Keplerian orbits [21]. A type I orbit defined in [21] will be used with the thrust vector directed along the $\mathbf{e}_Z$ axis. To determine the requirements for such orbits a frame of reference rotating with angular velocity $\boldsymbol{\omega}$ will be considered, as shown in Fig. 3. The dynamics of the problem can be investigated from Eq. (7) by transforming to the rotating frame such that

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 2\boldsymbol{\omega}\times\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = -\nabla V + \frac{1}{m_S}\boldsymbol{f} \tag{10}$$

Families of displaced circular orbits can be sought with fixed radius ρ, displacement distance z, and angular velocity ω corresponding to an equilibrium solution ($\dot{\mathbf{r}} = \ddot{\mathbf{r}} = 0$) in the rotating frame of reference, as shown in Fig. 3. Resolving along the $\mathbf{e}_Z$ and $\mathbf{e}_\rho$ axes then yields

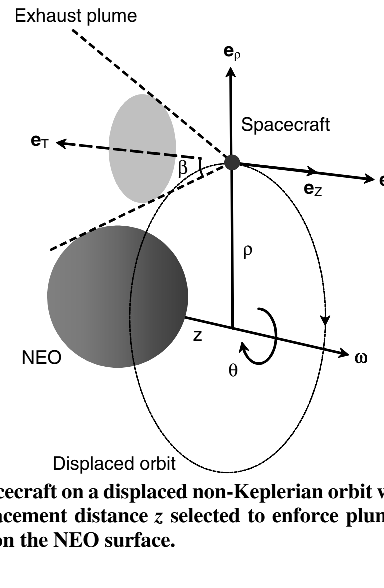

Fig. 3 Spacecraft on a displaced non-Keplerian orbit with orbit radius ρ and displacement distance z selected to enforce plume impingement constraints on the NEO surface.

$$-\omega^2 \rho \mathbf{e}_\rho = -\frac{Gm_N}{r^2}\left(\frac{\rho}{r}\right)\mathbf{e}_\rho - \frac{Gm_N}{r^2}\left(\frac{z}{r}\right)\mathbf{e}_z + \frac{1}{m_S} f \mathbf{e}_z \tag{11}$$

and so equating terms provides the required thrust f and orbital angular velocity ω as

$$\boldsymbol{f} = \frac{Gm_N m_S}{r^2}\left(\frac{z}{r}\right)\mathbf{e}_Z \tag{12a}$$

$$\omega = \sqrt{\frac{Gm_N}{r^3}} \tag{12b}$$

It can be seen from Eq. (12a) that the required thrust f is a function of ρ and z. For a fixed thrust, Eq. (12a) therefore defines a contour in the ρ–z plane. Each point on the contour corresponds to a displaced orbit with a given radius ρ, displacement distance z, and orbit period [obtained from Eq. (12b)]. Static equilibria are recovered when $\rho = 0$ and $z \neq 0$. Rather than a static equilibrium, the spacecraft orbit is now assumed to orbit the NEO on a displaced, non-Keplerian orbit. As will be seen, such an orbit can have significant advantages for the NEO deflection problem by satisfying thruster plume impingement constraints in certain circumstances. From Eq. (12a) it can be shown that the envelope of possible orbits for a fixed thrust is defined by

$$\rho = \left[\left(\frac{z}{\kappa}\right)^{2/3} - z^2\right]^{1/2}, \qquad \kappa = \frac{f}{Gm_N m_S} \tag{13}$$

as shown in Fig. 4. Finally, it can been shown that displaced non-Keplerian orbits are controllable using thrust modulation alone (while static equilibria are not) [21]. This represents a potentially simpler control strategy where the thrust vector direction can be fixed and its magnitude modulated to compensate for the inherent instability of the orbits. Active control of families of displaced non-Keplerian orbits has been discussed elsewhere [21,22], while the control of body-fixed hovering about NEOs with complex potentials has also been investigated [17].

Implementation and Evaluation

It has been noted that the use of a static equilibrium to couple the spacecraft and NEO is limited by plume impingement constraints [16]. In order for the center of mass of the spacecraft-NEO system to accelerate, a stream of reaction mass must escape and so transport momentum. Therefore, the exhaust plume must not impinge on the NEO surface. This requirement can be met by enforcing a constraint on the orientation of the spacecraft thrusters. It will now be assumed that the thrusters are symmetrically canted in direction $\mathbf{e}_T$ and $\mathbf{e}_{T'}$, as shown in Fig. 2. The orientation of the exhaust plume from the spacecraft can then be defined as $\cos\alpha = -\mathbf{e}_Z.\mathbf{e}_T$. In addition, it will be assumed that the exhaust plume can be represented by a uniform cone of half-angle β. For static equilibrium at a distance z from the center of mass of an NEO of radius R, it is clear from Fig. 2 that $\sin(\alpha - \beta) = R/z$. Therefore, the required thrust for static equilibrium defined in Eq. (9) is modified to become

$$\boldsymbol{f}_E(z) = \frac{Gm_S m_N}{z^2}\left[\cos\left(\sin^{-1}\left(\frac{R}{z}\right) + \beta\right)\right]^{-1}\mathbf{e}_Z \tag{14}$$

where the thrust vector would be decomposed along directions $\mathbf{e}_T$ and $\mathbf{e}_{T'}$. Clearly, the required thrust magnitude for the static equilibrium has been increased by the constraint on the orientation of the thrust vector to avoid plume impingement on the surface of the NEO. Although the detrimental effect of thruster canting is reduced for larger equilibrium distances (the NEO subtends a smaller solid angle), the available thrust magnitude also falls as z^{-2}. Smaller equilibrium distances allow a larger acceleration to be delivered to the center of mass of the spacecraft-NEO system, but at the expense of a rapidly increasing spacecraft thrust. It can be shown that the minimum equilibrium distance which satisfies the thruster plume impingement constraint is $z = R\sec\beta$, while $|\boldsymbol{f}_E| \to \infty$.

For a spacecraft orbiting on a displaced non-Keplerian orbit the plume impingement constraint can be met through an appropriate choice of the orbit radius and displacement distance, as shown in Fig. 3. The constraint can be determined through simple geometric analysis. In the ρ–z plane, the section of the thruster plume cone can be defined by the line $\rho = \eta z + Q$ (for some constants η and Q). In general, this line will twice intersect the section of the NEO in the ρ–z plane defined by circle $\rho^2 + z^2 = R^2$. The thruster plume will just intersect the NEO surface if the line element is tangent to the circle. This single intersection is found when the resulting quadratic equation defining the intersection has repeated, identical roots. Therefore, if the plume impingement constraint is met, the thrust vector can be directed along the $\mathbf{e}_Z$ axis without canting, allowing a more efficient transfer of momentum to the NEO than is available with a static equilibrium. From Eq. (12a) the required thrust is therefore

$$f_N(\rho, z) = \frac{Gm_N m_S z}{(\rho^2 + z^2)^{3/2}}\mathbf{e}_Z \tag{15}$$

To evaluate the use of displaced orbits for NEO deflection missions, the main parameters used in [6] will be adopted. It will be assumed that a 100 m NEO with a mean density of $2\ \mathrm{g \cdot cm^{-3}}$ is to be deflected using a large solar or nuclear electric propulsion vehicle with an initial mass of 20 metric tons. The plume impingement constraint will be set such that $\beta = 20$ deg. In [16] it was assumed that the spacecraft will hover in a close static equilibrium at a standoff distance of $z/R = 1.5$. However, numerical simulation shows that controlled hovering very near a small body while stable, can result in large (on the order of the small body size) oscillations in position [17]. Therefore, a more conservative standoff distance of $z/R = 2.5$ is adopted here, given the time scale over which hovering control is required. It should be noted, however, that larger standoff distances are more beneficial for the use of displaced orbits. For a plume impingement constraint set such that $\beta = 20$ deg, the minimum displaced orbit distance is found to be of order $z/R = 2.1$. For smaller displacement distances plume impingement constraints cannot be met, as will be seen below, and some thrust canting would be required.

For the above scenario ($z/R = 2.5$), the required thrust to be produced by the spacecraft for a static equilibrium can be determined from Eq. (14) and is found to be 0.247 N, corresponding to point A in Fig. 4. However, due to the required canting of the thrusters along directions $\mathbf{e}_T$ and $\mathbf{e}_{T'}$ to enforce plume impingement constraints, the effective thrust delivered to the center of mass of the spacecraft-NEO system along direction $\mathbf{e}_Z$ is only 0.179 N. If a displaced orbit is used, such that the orbit radius and displacement distance ensure that the plume impingement constraint is met ($\beta = 20$ deg), then the required thrust is provided by Eq. (15). For comparison, the thrust on

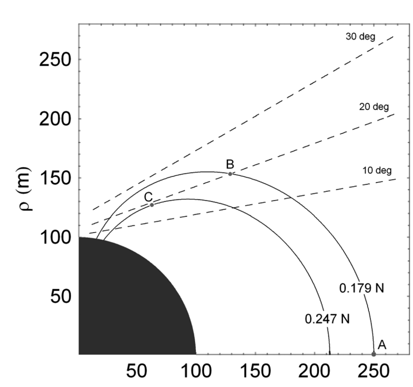

Fig. 4 Contours of required thrust for displaced non-Keplerian orbits (dashed lines: plume impingement constraints). A: static equilibrium with 0.247 N total thrust due to canted thrusters; B: displaced orbit with 0.179 N thrust and plume impingement constraint enforced; C: displaced orbit with 0.247 N thrust and plume impingement constraint not enforced.

the displaced orbit will be set at 0.179 N, corresponding to point B in Fig. 4. Therefore, both modes of coupling the spacecraft to the NEO will deliver the same net acceleration to the center of mass of the spacecraft-NEO system (and hence equivalent deflection Δv over a given interval). However, coupling using the displaced non-Keplerian orbit requires significantly less thrust (~25%) to be developed by the spacecraft itself. Alternatively, for a spacecraft sized to develop a thrust of 0.247 N, the full thrust could be directed along the $\mathbf{e}_Z$ axis if a displaced orbit is used. However, it can be seen from Fig. 4 (point C) that the plume impingement constraint is not quite met in this case, illustrating the limitations noted above.

Finally, for spacecraft stationed extremely close to the NEO, displaced orbits can still be used but with the spacecraft thrust vector canted toward the axis of symmetry of the orbit (type III orbits defined in [21]). Here the component of thrust along the $\mathbf{e}_Z$ axis is diminished, whereas the component of thrust now induced along the $\mathbf{e}_\rho$ axis is balanced by selecting an orbit angular velocity which will generate sufficient centripetal force to balance this component of thrust. For the case $z/R = 1.5$ considered in [16], the static equilibrium requires a total spacecraft thrust of 1.051 N, with a useful thrust 0.497 N delivered along the $\mathbf{e}_Z$ axis due to thruster canting to avoid plume impingement. A displaced orbit can also be used with the thrust vector canted inward toward the axis of symmetry of the orbit to avoid thruster plume impingement. A spacecraft with the same total thrust of 1.051 N can orbit with $z = 130$ m, $\rho = 50$ m and with the thrust vector canted inward 60 deg toward the axis of symmetry of the orbit, which ensures that the thrust plume does not impinge on the NEO surface. The useful thrust directed along the $\mathbf{e}_Z$ axis is then 0.526 N, which is still somewhat more efficient than the static equilibrium.

Conclusions

The dynamics of a gravitationally bound spacecraft-NEO system has been investigated and it has been demonstrated from first principles that the center of mass of the spacecraft-NEO system will accelerate. In addition, it has been shown that displaced non-Keplerian orbits can provide a more advantageous means of coupling the spacecraft and NEO than a static equilibrium in certain circumstances. For appropriate orbit parameters, the spacecraft thrusters do not need to be canted, as in the case for a static equilibrium where plume impingement on the NEO is a key issue. The use of displaced non-Keplerian orbits can therefore allow a smaller thrust magnitude to be used to effect the same deflection of the NEO, provided the displacement distance is large enough.

References

[1] Alvarez, L. E., Alvarez, W., Asaro, F., and Michel, H. V., "Extra-Terrestrial Cause for the Cretaceous-Tertiary Extinction," *Science*, Vol. 208, No. 4448, 1980, pp. 1095–1108.

[2] Chapman, C. R., and Morrison, D., "Impact on the Earth by Asteroids and Comets: Assessing the Hazard," *Nature (London)*, Vol. 367, No. 6458, 1994, pp. 33–39.

[3] Binzel, R. P., "The Torino Impact Hazard Scale," *Planetary and Space Science*, Vol. 48, No. 4, 2000, pp. 297–303.

[4] Rabinowitz, D., Helin, E., Lawrence, K., and Pravdo, S., "A Reduced Estimate of the Number of Kilometre-Sized Near-Earth Asteroids," *Nature (London)*, Vol. 403, No. 6766, 2000, pp. 165–166.

[5] Ahrens, T. J., and Harris, A. W., "Deflection and Fragmentation of Near-Earth Asteroids," *Nature (London)*, Vol. 360, No. 6403, 1992, pp. 429–433.

[6] Melsoh, H. J., "Solar Asteroid Diversion," *Nature (London)*, Vol. 366, No. 6450, 1993, pp. 21, 22.

[7] Melosh, H. J., Nemchinov, I. V., and Zetzer, Y. I., "Non-Nuclear Strategies for Deflecting Comets and Asteroids," *Hazards due to Comets and Asteroids*, edited by T. Gehrels, Univ. of Arizona Press, Tucson, AZ, 1994, pp. 1111–1132.

[8] Spitale, J. N., "Asteroid Hazard Mitigation Using the Yarkovsky Effect," *Science*, Vol. 296, 2002, p. 77.

[9] McInnes, C. R., "Deflection of Near-Earth Asteroids by Kinetic Energy Impacts from Retrograde Orbits," *Planetary and Space Science*, Vol. 52, No. 7, 2004, pp. 587–590.

[10] Dachwald, B., and Wie, B., "Solar Sail Trajectory Optimization for Intercepting, Impacting, and Deflecting Near-Earth Asteroids," AIAA Paper 2005-6176, Aug. 2005.

[11] Sagan, C., and Ostro, S., "Dangers of Asteroid Deflection," *Nature (London)*, Vol. 368, No. 6471, 1994, p. 501.

[12] Carusi, A., Valsecchi, G., D'Abramo, G., and Boattini, A., "Deflecting NEOs in Route of Collision with the Earth," *Icarus*, Vol. 159, No. 2, 2002, pp. 417–422.

[13] Izzo, D., Bourdoux, A., Walker, R., and Ongaro, F., "Optimal Trajectories for the Impulsive Deflection of NEOs," IAC Paper 05-C1.5.06, 2005.

[14] Walker, R., Izzo, D., de Negueruela, C., Summerer, L., Ayre, M., and Vasile, M., "Concepts for Near Earth Asteroid Deflection Using Spacecraft with Advanced Nuclear and Solar Electric Propulsion Systems," IAC Paper 04-Q.5.08, Oct. 2004.

[15] Scheeres, D., and Schweickart, R., "The Mechanics of Moving Asteroids," AIAA Paper 2004-1446, Feb. 2004.

[16] Lu, E. T., and Love, S. G., "Gravitational Tractor for Towing Asteroids," *Nature (London)*, Vol. 438, No. 7065, 2005, pp. 177–178.

[17] Broschart, S. B., and Scheeres. D. J., "Control of Hovering Spacecraft Near Small Bodies: Application to Asteroid 25143 Itokawa," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 343–354.

[18] Asphaug, E., Ostro, S. J., Hudson, R. S., Scheeres, D. J., and Benzi, W., "Disruption of Kilometre-Sized Asteroids by Energetic Collisions," *Nature (London)*, Vol. 393, No. 6684, 1998, pp. 437–440.

[19] Shkadov. L. M., "Possibility of Controlling Solar System Motion in the Galaxy," IAA Paper 87-613, Oct. 1987.

[20] McInnes, C. R., "Astronomical Engineering Revisited: Planetary Orbit Modification Using Solar Radiation Pressure," *Astrophysics and Space Science*, Vol. 282, No. 4, 2002, pp. 765–772.

[21] McInnes, C. R., "Dynamics, Stability and Control of Displaced Non-Keplerian Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 5, 1998, pp. 799–805.

[22] Bookless, J., and McInnes, C. R., "Dynamics and Control of Displaced Periodic Orbits Using Solar Sail Propulsion," *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 3, 2006, pp. 527–537.